Section 4.1

Angle
An angle is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the initial side of the angle, and the position after rotation is the terminal side. The endpoint of the ray is the vertex of the angle.
Positive angles are generated by counterclockwise rotation, and negative angles by clockwise rotation.

Coterminal Angles
If two angles have the same initial and terminal sides, such angles are coterminal.

Radian
One radian is the measure of a central angle θ that intercepts and arc s equal in length to the radius r of the circle. Algebraically, this means $\theta = \frac{s}{r}$, where θ is measured in radians.

Conversions Between Radians and Degrees

a) To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$.

b) To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.

Arc Length
For a circle of radius r, a central angle θ intercepts an arc of length s given by $s = r\theta$, where θ is measured in radians. Note that is $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Area of a Sector of a Circle
For a circle of radius r, the area A of a sector of the circle with central angle θ is given by $A = \frac{1}{2}r^2\theta$, where θ is measured in radians.

Linear and Angular Speeds
Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the linear speed v of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the angular speed ω of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

Acute and Obtuse Angles

Angles between 0 and $\pi/2$ are acute angles, and angles between $\pi/2$ and π are obtuse angles.

Complimentary and Supplementary Angles

Two positive angles α and β are complementary if their sum is $\pi/2$. Two positive angles are supplementary if their sum is π.

Problem 1. Determine the quadrant in which each angle lies.

a) $\theta = 120°, 300°, -280°, 250°$

b) $\theta = \frac{4}{9}\pi,\ -5.75,\ \frac{2}{3}\pi,\ 2.$

Problem 2. Determine two coterminal angles (one positive and one negative) for each angle.

a) $\theta = -42°$

b) $\theta = \frac{7\pi}{8}$

Problem 3. Find (if possible) the complement and supplement of each angle.

a) $28°$

b) $145°$

c) $\frac{\pi}{6}$

d) $\frac{7\pi}{8}$

Problem 4. Rewrite each angle in radian measure as a multiple of π.

a) $330°$

b) $30°$

c) $-90°$

d) $235°$

Problem 5. Rewrite each angle in degree measure.

a) $-\frac{5\pi}{6}$

b) $\frac{\pi}{10}$

c) $\frac{7\pi}{4}$

Problem 6. Convert the angle measure from degrees to radians.

a) $126.5°$

b) $255°$

Problem 7. Convert the angle measure from radians to degrees.

a) $\frac{7\pi}{9}$

b) 2.3π

Problem 8. Find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.

a) Radius $r = 8$ feet, arc length $s = 5$ feet.

b) Radius $r = 120$ km, arc length $s = 250$ km.

Problem 9. Find the length of the arc on a circle of radius r intercepted by a central angle θ.

a) Radius $r = 7$ feet, $\theta = 30°$.

b) Radius $r = 45$ cm, $\theta = \frac{7\pi}{4}$.

Problem 10. Find the area of the sector of the circle with radius r and central angle θ.

a) Radius $r = 25$ millimeters, $\theta = \frac{\pi}{6}$.

b) Radius $r = 2.6$ miles, $\theta = 240°$.

Problem 11. A Ferris wheel with a diameter of 200 feet rotates 3 times per minute.

a) Find the angular speed of the Ferris wheel.

b) Find the linear speed of one of the chairs on the Ferris wheel.